Rapid Sizing Method for Airplanes

Jan Roskam*
The University of Kansas, Lawrence, Kansas

A method for rapidly determining airplane takeoff weight, empty weight, and mission fuel weight for a given mission specification is presented. The method applies to a wide range of airplane types and relies on statistical relations between empty weight and takeoff weight of production airplanes. The method can also be used to predict the sensitivity of takeoff weight with respect to changes in the mission specifications as well as to changes in airplane lift-to-drag ratio, specific fuel consumption, and/or propeller efficiency.

Nomenclature

A,B = regression coefficients in Eq. (7)
C = coefficient in Eq. (18), defined in Eq. (19)
c_j = engine specific fuel consumption (jet), lb/lb/h
c_p = engine specific fuel consumption (prop), lb/hp/h
D = sum of payload weight and crew weight, see Eq. (5), lb
E_{ltr} = endurance during loiter, h
F = constant defined in Eq. (28), lb
L/D = lift-to-drag ratio
M_{res} = reserve fuel weight fraction, see Eq. (9)
M_{tfo} = trapped fuel and oil weight fraction, often 0.005
M_{used} = used fuel weight fraction, see Eqs. (10) and (11)
M'_{ff} = see Eq. (12)
R_{cr} = cruise range, sm (statute miles)
V = speed (cruise or loiter), smph (statute miles per hour)
W_E = empty weight, lb
W_F = mission fuel weight carried, lb
$W_{F_{res}}$ = mission fuel reserves, lb
$W_{F_{used}}$ = mission fuel weight used, lb
W_i = weight at start of mission phase i, lb
W_{i+1} = weight at end of mission phase i, lb
W_{OE} = operating empty weight
W_{PL} = payload weight, lb
W_{TO} = takeoff weight, lb
Δ = function defined in Eq. (25)
η_p = propeller efficiency
Π = product of

Introduction

A METHOD is presented herein with the objective to make possible the rapid "weight sizing" of airplanes that must meet an arbitrary mission specification. Weight sizing is defined as the determination of minimum required takeoff weight, W_{TO}, empty weight, W_E, and mission fuel weight, W_F, for a given mission specification.

It will be shown that not all mission specifications result in realizable airplanes: the sizing method does not always converge. A "test" for determining convergence or lack thereof is also presented.

The method can also be used to determine the sensitivity of airplane takeoff weight to performance requirements (as stated in the mission specification) as well as to aerodynamic and propulsive efficiencies.

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*Ackers Distinguished Professor of Aerospace Engineering. Associate Fellow AIAA.

Outline of the Method

It is assumed that a mission specification for an airplane that needs to be "sized" is available. Table 1 contains an example of such a mission specification. Figure 1 provides an example of a corresponding mission profile with all mission phases numbered.

Airplane takeoff weight is defined as follows:

$$W_{TO} = W_{OE} + W_F + W_{PL} \tag{1}$$

and the operating empty weight in turn is defined as

$$W_{OE} = W_E + W_{tfo} + W_{crew} \tag{2}$$

The following quantities are introduced: M_{tfo} is the "trapped fuel and oil" fraction, defined by

$$W_{tfo} = M_{tfo} W_{TO} \tag{3}$$

For most airplanes the trapped fuel and oil fraction is a small number. Typically,

$$M_{tfo} = 0.001 \text{ to } 0.005 \tag{4}$$

D is the sum of payload and crew weight:

$$D = W_{PL} + W_{crew} \tag{5}$$

Values for payload weight and crew weight normally follow from the mission specification.

Combining Eqs. (1-3) and (5) yields

$$W_{TO} = W_E + W_F + D + M_{tfo} W_{TO} \tag{6}$$

It is shown in Ref. 1 that the following statistical relation exists between W_{TO} and W_E:

$$\log_{10} W_E = \{(\log_{10} W_{TO}) - A\}/B \tag{7}$$

The statistically determined regression coefficients A and B are defined in Table 2 for 12 types of airplanes. These numbers reflect state-of-the-art design and production practice. Figures 2 and 3 show examples of two such correlations.

The mission fuel, W_F in Eq. (6), can be broken down as follows:

$$W_F = W_{F_{res}} + W_{F_{used}} \tag{8}$$

The fuel reserve weight, $W_{F_{res}}$, is defined as

$$W_{F_{res}} = M_{res} W_{F_{used}} \tag{9}$$

The fuel used, $W_{F_{\text{used}}}$, can be found from

$$W_{F_{\text{used}}} = M_{\text{used}} W_{\text{TO}} \tag{10}$$

where the "used fuel" weight fraction, M_{used}, is defined as

$$M_{\text{used}} = (1 - M'_{ff}) \tag{11}$$

The following discussion shows how M_{res} and M'_{ff} may be determined.

The mission specification normally contains the information from which the ground rules that define the required amount of fuel reserves can be delineated. Fundamentally, two ways exist to account for fuel reserves:

1) The fuel reserves are given as a fraction of the fuel used. In this case, the fuel reserve weight fraction, M_{res}, is specified in the mission specification as some fraction between 0 and 1.0. This fraction needs to be substituted in Eq. (9) in this case.

2) The fuel reserves are specified as an extension of the mission profile. In this case, the mission definition is augmented by the requirement to fly to an alternate airport and/or to loiter for specified amounts of time. The reserve mission phases may be added to the mission profile shown in Fig. 1, resulting in the overall mission profile of Fig. 4.

Therefore, $M_{\text{res}} = 0$ should be substituted into Eq. (9), because $W_{F_{\text{used}}}$ now represents mission fuel + reserve fuel.

The quantity M'_{ff} in Eq. (11) may be determined from

$$M'_{ff} = \frac{W_1}{W_{\text{TO}}} \prod_{i=1}^{i=n} \frac{W_{i+1}}{W_i} \tag{12}$$

The weight fractions W_1/W_{TO} and W_{i+1}/W_i may be estimated from Table 3 *except for those mission phases that are very fuel intensive, such as cruise, loiter, and/or fly to an alternate*. For the latter mission phases (phase 5 in Fig. 1 and phases 5, 8, and 9 in Fig. 4), the corresponding weight fractions should be estimated with the following Breguet equations [Eqs. (13-16)]:

1) For cruise mission phases:
For propeller-driven airplanes:

$$R_{\text{cr}} = 375(\eta_p/c_p)_{\text{cr}}(L/D)_{\text{cr}} \ln(W_4/W_5)$$

$$[R_{\text{cr}} \text{ is in statute miles (sm)!}] \tag{13}$$

For jet-driven airplanes:

$$R_{\text{cr}} = (V/c_j)(L/D)_{\text{cr}} \ln(W_4/W_5)$$

$$[\text{if } V \text{ is in statute miles per hour (smph), } R \text{ is in sm!}] \tag{14}$$

2) For loiter mission phases:
For propeller-driven airplanes:

$$E_{\text{ltr}} = 375(1/V)_{\text{ltr}}(\eta_p/c_p)_{\text{ltr}}(L/D)_{\text{ltr}} \ln(W_5/W_6)$$

$$[V \text{ in smph!}] \tag{15}$$

For jet-driven airplanes:

$$E_{\text{ltr}} = (1/c_{j_{\text{ltr}}})(L/D)_{\text{ltr}} \ln(W_5/W_6) \tag{16}$$

Typical values for η_p, c_p, c_j, and L/D may be found in Table 4 for each particular mission phase. However, if actual

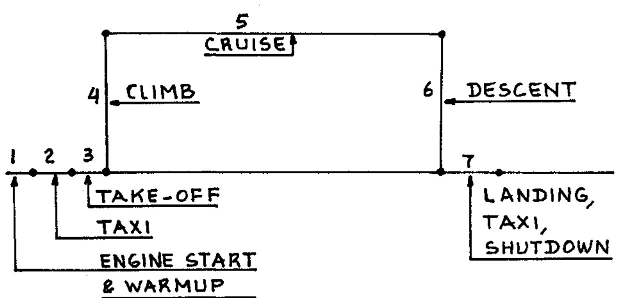

Fig. 1 Mission profile for airplane of Table 1.

Table 1 Mission specification for a twin-engine propeller-driven airplane

Payload:	Six passengers at 175 lb each (including the pilot) and 200 lb total baggage
Range:	1000 sm with maximum payload
Reserves:	25% of mission fuel
Altitude:	10,000 ft for the design range
Cruise speed:	250 knots at 75% power at 10,000 ft and at takeoff weight
Climb:	10 min to 10,000 ft at takeoff weight
Takeoff and landing	1500 ft ground run at sea level, standard day. Landing at 0.95 of takeoff weight
Powerplants:	Piston/propeller
Certification base:	FAR 23

Table 2 Regression coefficients A and B for Eq. (7)

Airplane type	A	B
Homebuilts		
Personal fun	0.3411	0.9519
Scaled fighters	0.5542	0.8654
Composites	0.8222	0.8050
Single-engine piston/props	−0.1440	1.1162
Twin-engine props	0.0966	1.0298
Agricultural	−0.4398	1.1946
Business jets	0.2678	0.9979
Regional turboprops	0.3774	0.9647
Transport jets	0.0833	1.0383
Military trainers		
Jets	0.6632	0.8640
Turbo/props	0.1677	0.9978
Piston/props	0.5627	0.8761
Fighters		
Jets (incl. ext. load)	0.5091	0.9505
Jets (clean)	0.1362	1.0116
Turboprops	0.2705	0.9830
Military patrol, bomb, and transport		
Jets	−0.2009	1.1037
Turboprops	−0.4179	1.1446
Flying boats, amphibious, and float airplanes	0.1703	1.0083
Supersonic cruise airplanes	0.4221	0.9876

data for these quantities are available, they should be used instead of referring to Table 4.

Important notes:

1) In some airplanes the weight at engine start (see phase 1 in Fig. 1) is in fact the *ramp weight* and not the takeoff weight. To calculate the takeoff weight properly for those airplanes, the first three weight fractions, W_1/W_{TO}, W_2/W_1, and W_3/W_2, should be omitted from Eq. (12), which becomes

$$M'_{ff}=\frac{W_4}{W_{TO}}\prod_{i=4}^{i=n}\frac{W_{i+1}}{W_i} \tag{17}$$

2) Typical ramp weight to takeoff weight ratios are between 1.005 and 1.010. The effect of neglecting this is usually small.

At this point all of the information needed to determine airplane takeoff weight, W_{TO}, empty weight, W_E, and mission fuel weight, W_F, is available. Combining Eqs. (6-12) yields Eq. (18) from which the takeoff weight, W_{TO}, may be computed:

$$\log_{10}(W_{TO})=A+B\log_{10}(CW_{TO}-D) \tag{18}$$

where the quantity C is defined as

$$C=1-(1+M_{res})M_{used}-M_{tfo} \tag{19}$$

The empty weight, W_E, can be found from

$$W_E=CW_{TO}-D \tag{20}$$

and the mission fuel weight, W_F, is

$$W_F=W_{TO}-W_E-D-W_{tfo} \tag{21}$$

Application of the Method

An example illustrating how to use the method will be given as a step-by-step procedure:

Step 1. From the airplane mission specification, construct a mission profile and number of mission phases.—Table 1 lists an example mission specification and Fig. 1 shows the corresponding mission profile.

Step 2. From the mission specification determine the airplane type in Table 2 that most closely represents the airplane being designed.—The example airplane in Table 1 is a type 3 airplane in Table 2.

Step 3. From Table 2 find the values of the regression coefficients A and B.—For type 3 airplanes, Table 2 yields: $A=0.0966$ and $B=1.0298$.

Step 4. From the mission specification find the values for crew weight and payload weight.—From Table 1 and Eq. (5): $D=(6\times 175)+200=1250$ lb. Note: In this type of airplane the crew weight is frequently counted as part of the payload.

Step 5. From the mission specification determine the requirements for reserve fuel.—From Table 2 it follows that, in this case, $W_{F_{res}}=0.25W_{F_{used}}$ (i.e., $M_{res}=0.25$).

Step 6. From the mission specification ascertain whether or not this airplane has a ramp weight, W_{ramp}, which differs significantly from the takeoff weight, W_{TO}.—For the example airplane the ramp weight question will be neglected.

Step 7. Estimate the weight ratios W_{i+1}/W_i in Eq. (12) from Table 3, or from Eqs. (13-16).—Table 5 lists these weight ratios for the example airplane of Table 1. From Eq. (12) the value of M'_{ff} is

$$M'_{ff}=0.992\times0.996\times0.996\times0.990$$
$$\times0.863\times0.992\times0.992=0.827$$

Note: For military airplanes that drop part of the payload during the mission (for example, by release of stores) it is necessary to correct the weight fractions in Eq. (12). Reference 1 shows how this may be done.

Step 8. Compute C from Eq. (19)

$$C=\{1-(1+0.25)(1-0.827)-0.005\}=0.779$$

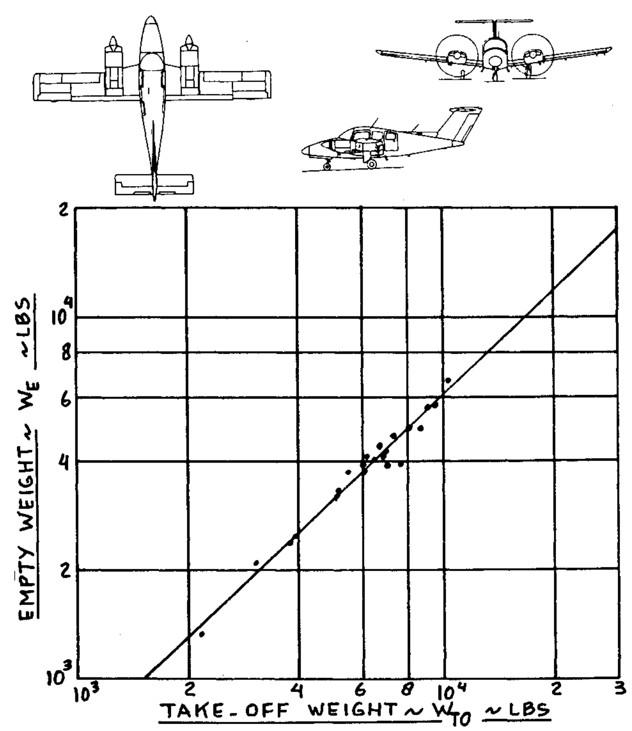

Fig. 2 Takeoff-weight and empty-weight correlation for twin-engine, propeller-driven airplanes.

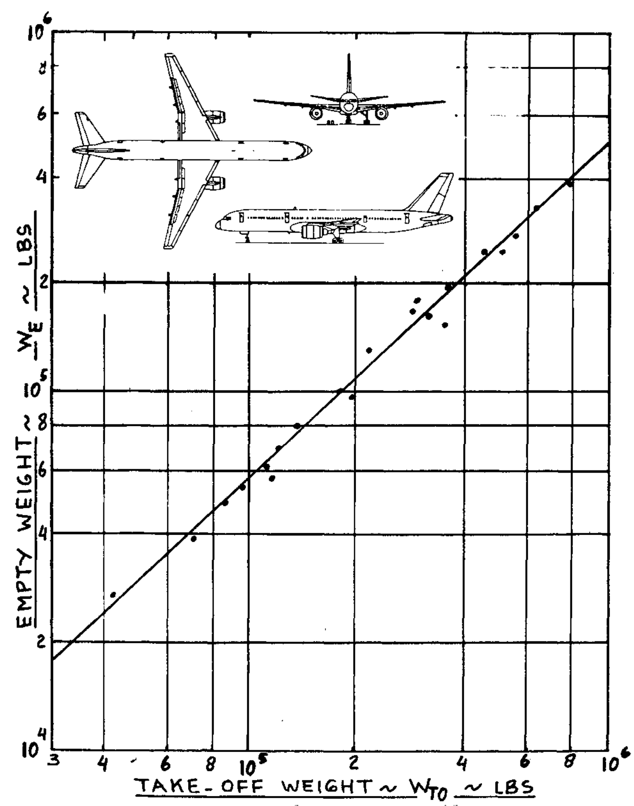

Fig. 3 Takeoff-weight and empty-weight correlation for transport jets.

Table 3 Suggested weight fractions for several mission phases (Fig. 1)

Airplane type	Engine start, warmup, phase 1	Taxi phase 2	Take off, phase 3	Climb phase 4	Descent, phase 7	Landing, taxi, shutdown, phase 8
Homebuilts	0.998	0.998	0.998	0.995	0.995	0.995
Single engine piston/props	0.995	0.997	0.998	0.992	0.993	0.993
Twin-engine props	0.992	0.996	0.996	0.990	0.992	0.992
Agricultural	0.996	0.995	0.996	0.998	0.999	0.998
Business jets	0.990	0.995	0.995	0.980	0.990	0.992
Regional turboprops	0.990	0.995	0.995	0.985	0.985	0.995
Transport jets	0.990	0.990	0.995	0.980	0.990	0.992
Military trainers	0.990	0.990	0.990	0.980	0.990	0.995
Fighters	0.990	0.990	0.990	0.96-0.90	0.990	0.995
Military patrol, bomb, and transports	0.990	0.990	0.995	0.980	0.990	0.992
Flying boats, amphibious and float	0.992	0.990	0.996	0.985	0.990	0.990
Supersonic cruise	0.990	0.995	0.995	0.92-0.87	0.985	0.992

Note: Numbes are averages based on experience. If better data are available, they should be used!

Table 4 Suggested values for L/D, c_j, c_p, and η_p for several mission phases (Fig. 1)

	Cruise, phase 5				Loiter, phase 6			
Airplane type	L/D	c_j	c_p	η_p	L/D	c_j	c_p	η_p
Homebuilts	8-10	—	0.6-0.8	0.7	10-12	—	0.5-0.7	0.6
Single-engine piston/props	8-10	—	0.5-0.7	0.8	10-12	—	0.5-0.7	0.7
Twin-engine props	8-10	—	0.5-0.7	0.82	9-11	—	0.5-0.7	0.72
Agricultural	5-7	—	0.5-0.7	0.82	8-10	—	0.5-0.7	0.72
Business jets	10-12	0.5-0.9	—	—	12-14	0.4-0.6	—	—
Regional turboprops	11-13	—	0.4-0.6	0.85	14-16	—	0.5-0.7	0.77
Transport jets	13-15	0.5-0.9	—	—	14-18	0.4-0.6	—	—
Military trainers	8-10	0.5-1.0	0.4-0.6	0.82	10-14	0.4-0.6	0.5-0.7	0.77
Fighters	4-7	0.6-1.4	0.5-0.7	0.82	6-9	0.6-0.8	0.5-0.7	0.77
Military patrol, bomb, and transports	13-15	0.5-0.9	0.4-0.7	0.82	14-18	0.4-0.6	0.5-0.7	0.77
Floating boats, amphibious, and float airplanes	10-12	0.5-0.9	0.5-0.7	0.82	13-15	0.4-0.6	0.5-0.7	0.77
Supersonic cruise	4-6	0.7-1.5	—	—	7-9	0.6-0.8	—	—

Note: Numbers are averages based on experience. If better data are available, they should be used!

Table 5 Calculation of quantity M_{ff} of Eq. (12) for the example airplane of Table 1

Mission phase	Phase no.	Weight fraction	Range	L/D	c_p	η_p
Engine start and warmup	1	$W_1/W_{TO}=0.992$[a]				
Taxi	2	$W_2/W_1=0.996$[a]				
Takeoff	3	$W_3/W_2=0.996$[a]				
Climb	4	$W_4/W_3=0.990$[a]				
Cruise	5	$W_5/W_4=0.863$ Eq. (13)	1000[b]	11.0[c]	0.5[c]	0.82[c]
Descent	6	$W_6/W_5=0.992$[a]				
Landing, taxi, shutdown	7	$W_7/W_6=0.992$[a]				

[a]From Table 3. [b]From Table 1. [c]From Table 4.

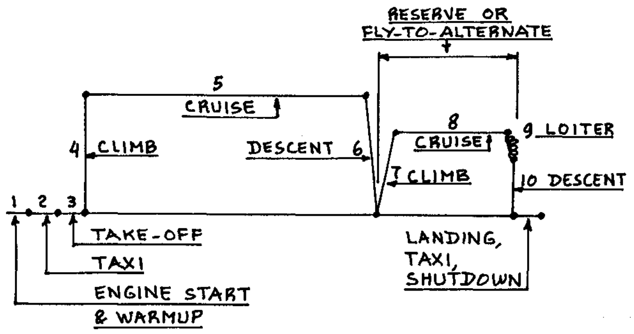

Fig. 4 Example of mission profile extended with flight to alternate.

Step 9. Compute W_{TO} from Eq. (18) with

$A=0.0966$, $B=1.0298$, $C=0.779$, and $D=1250$ lb

(The answer is $W_{TO}=7900$ lb)

Step 10. Compute W_E from Eq. (20),

$$W_E=0.779\times 7900-1250=4904 \text{ lb}$$

Step 11. Compute W_F from Eq. (21),

$$W_F=7900-4904-1250-0.005\times 7900=1706 \text{ lb}$$

Note that the fuel used during this mission follows from Eqs. (10) and (11) as

$$W_{F_{\text{used}}}=0.173\times 7900=1367 \text{ lb}$$

The next section presents a discussion of the convergence properties of this sizing method.

Convergence Properties of this Method

The convergence of this sizing method reduces to the question: Does Eq. (18) always have a solution? The behavior of the functions $\log W_{TO}$ and $\{A+B\log(CW_{TO}-D)\}$ when plotted against W_{TO} is crucial. Figure 5 illustrates the behavior of these functions for four example airplanes.

What is critical is the difference function Δ, defined as

$$\Delta=(\log W_{TO})-A-B\log(CW_{TO}-D) \tag{22}$$

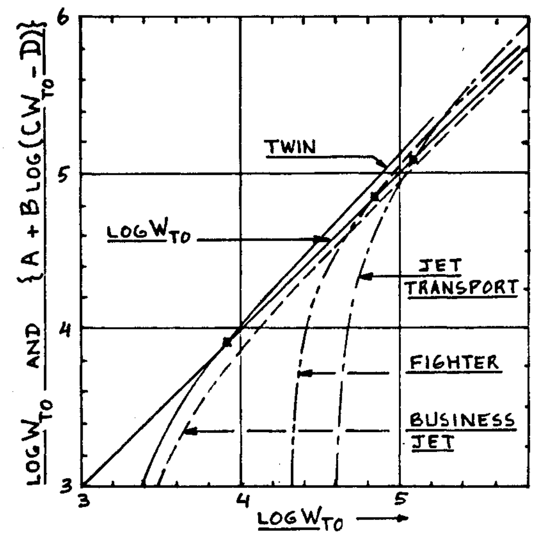

Fig. 5 Illustration of convergence properties of Eq. (18).

There are three possibilities for this function, Δ:

1) Δ has no minimum or maximum value. In this case one solution for takeoff weight will exist. In Fig. 5 this means that the lines $\log W_{TO}$ and $\{A+B\log(CW_{TO}-D)\}$ intersect at one point.

2) Δ has a negative minimum value. In this case there are two solutions to takeoff weight. In Fig. 5 this means that the lines $\log W_{TO}$ and $\{A+B\log(CW_{TO}-D)\}$ intersect at two points. Only the lowest value of W_{TO} is of practical interest.

3) Δ has a positive minimum value. In this case there is no value for takeoff weight, which satisfies Eq. (18). In Fig. 5 this means that the lines $\log W_{TO}$ and $\{A+B\log(W_{TO}-D)\}$ do not intersect.

Whichever is the case can be ascertained by differentiation of Δ with respect to W_{TO} and setting the result equal to zero:

$$\frac{\partial\Delta}{\partial W_{TO}}=0=\frac{1}{W_{TO}}-\frac{BC}{CW_{TO}-D} \tag{23}$$

From this it is found that

$$W_{TO}=D/C(1-B) \tag{24}$$

Since D and C are always positive an optimum does not exist for: $B>1$. In this case a solution always exists: Case 1.

Table 6 Example of convergence characteristics for four airplanes

	Twin	Jet	Fighter	Business jet
A	0.0966	0.0833	0.5091	0.2678
B	1.0298	1.0383	0.9595	0.9979
C	0.779	0.791	0.708	0.50
D	1250	31,775	12,200	1000 (D, lb)
	$B>1$	$B>1$	$B<1$	$B<1$
	Δ does not exist		$\Delta=-0.071$ $\Delta<0$	$\Delta=0.046$ $\Delta>0$
	1 solution	1 solution	2 solutions	no solution(s)

An optimum does exist for $B<1$. Whether or not this optimum represents case 2 or 3 depends on the sign of Δ at the value of W_{TO} found from Eq. (24).

Substitution of Eq. (24) into Eq. (22) yields

$$\Delta=\log\{D/C(1-B)\}-A-B\log\{DB/(1-B)\} \tag{25}$$

If $\Delta<0$ there are two solutions: Case 2. If $\Delta>0$ there is no solution (no convergence): Case 3.

Table 6 shows how Δ behaves for four example airplanes. Note that in one case, no convergence occurs.

It would be reasonable to expect that all airplane types could run into a mission definition for which no convergence occurs (for example, by demanding an unrealistically large value of range). This analysis suggests that lack of convergence will occur only for airplane types with $B<1$ and $\Delta>0$. Realistically this does not seem to make sense! The following explanation is offered:

The value B merely represents a statistically obtained number which represents the slope of the $\log W_{TO}$ vs $\log W_E$ data for airplanes with similar mission orientations. It should be expected that these statistical correlations *are valid only for weight ranges not too far outside the ranges for which the correlations were obtained in the first place.* Outside these ranges it should be expected that the slopes will change so that convergence will, in fact, not occur for missions that would require very high weights: at the gross weights outside the correlation range the airplane should become less structurally efficient.

The next section deals with the question of takeoff weight sensitivity to changes in the mission specification and in parameters such as L/D, c_p, c_j, and η_p.

Sensitivity Analyses

The following questions always arise during the preliminary sizing process.

How sensitive is the takeoff weight of an airplane as a result of making changes in: 1) The mission specification: range, R; endurance, E; and mission reserves, M_{res}. 2) Aerodynamic and propulsive efficiencies: L/D, c_p, c_j, and η_p.

It is shown shown in Ref. 1 that these sensitivities can all be obtained with the help of a relatively simple process of partial differentiation. Because of Eqs. (7) and (18) it is possible to find the following explicit expressions for airplane takeoff weight sensitivities.

Sensitivities to Items in the Mission Specification

Sensitivity of takeoff weight, W_{TO}, to the following items in the mission specification will be considered: payload weight, W_{PL}, and range, R.

In Ref. 1 equations for sensitivity of takeoff weight to these parameters are derived for jet- and propeller-driven airplanes. For the example airplane used herein (a general-aviation propeller-driven twin) these equations are:

$$\frac{\partial W_{TO}}{\partial W_{PL}}=BW_{TO}\{D-C(1-B)W_{TO}\}^{-1} \tag{26}$$

$$\frac{\partial W_{TO}}{\partial R}=Fc_p(375\eta_p L/D)^{-1} \tag{27}$$

where

$$F=-BW_{TO}^2\{CW_{TO}(1-B)-D\}^{-1}(1+M_{res})M_{used} \tag{28}$$

For the example airplane the following values are obtained:

$$\frac{\partial W_{TO}}{\partial W_{PL}}=5.7 \text{ lb/lb} \quad \text{and} \quad \frac{\partial W_{TO}}{\partial R}=6.9 \text{ lb/n.mi.}$$

These results have the following meaning:

To carry one additional pound of payload while keeping the other items in the mission specification constant requires an increase of 5.7 lb in takeoff weight. To increase range by 1 n.mi. while keeping the other items in the mission specification constant requires an increase of 6.9 lb in takeoff weight.

Sensitivities to Aerodynamic and Propulsive Parameters

Equations for the sensitivities of airplane takeoff weight to aerodynamic and propulsive parameters are derived in Ref. 1 for jet- and propeller-driven airplanes. For the propeller-driven example airplane these equations are:

$$\frac{\partial W_{TO}}{\partial (L/D)}=-FRc_p\{375\eta_p(L/D)^2\}^{-1} \tag{29}$$

$$\frac{\partial W_{TO}}{\partial c_p}=FR(375\eta_p \; L/D)^{-1} \tag{30}$$

$$\frac{\partial W_{TO}}{\partial \eta_p}=-FRc_p(375\eta_p^2 L/D)^{-1} \tag{31}$$

For the example general-aviation propeller-driven twin, the values for these sensitivities are found to be

$$\frac{\partial W_{TO}}{\partial (L/D)}=-628 \text{ lb}$$

$$\frac{\partial W_{TO}}{\partial c_p}=13{,}897 \text{ lb/(lb/hp/h)}$$

$$\frac{\partial W_{TO}}{\partial \eta_p}=-8425 \text{ lb}$$

These sensitivities have the following meaning:

If the L/D of the airplane is increased from 11.0 to 12.0 keeping all items in the mission specification the same, the takeoff weight will decrease by 628 lb.

If the specific fuel consumption of the engine in cruise is changed from 0.5 to 0.6 (an increase!), while keeping all items in the mission specification the same, the takeoff weight would increase by 1390 lb.

If the propeller efficiency in cruise were increased from 0.82 to 0.85, while keeping all items in the mission specification the same, the takeoff weight would decrease by $0.03\times 8425=253$ lb.

Concluding Remarks

It has been shown that the process of preliminary airplane weight sizing for an arbitrary mission specification can be carried out rapidly and without sophisticated computer programs. It has also been shown that the sensitivity of airplane takeoff weight to changes in mission performance and/or aerodynamic and propulsion efficiencies can be computed from simple, closed-form equations. It is thus possible to conduct trade studies of these effects without having designed a "baseline" airplane. The method should be particularly useful to aeronautical engineers in arriving quickly at ball-park designs.

Reference

[1]Roskam, J., *Airplane Design, Part I: Preliminary Sizing of Airplanes*, Roskam Aviation and Engineering Corp., Ottawa, KS, pp. 5-88.